Contents

Introduction

0.1 Why is mathematics important

To start a career in one of the following disciplines you will need mathematics:

- Business
- Science
- Engineering Technology
- Information Technology
- Architecture
- Visual Art (Leonardo da Vinci, 3D animations, games)

Mathematics is where the money is.

☞ **It is easier to make money through mathematics than trying to become a superstar.**

0.2 Positive attitude

- Anyone can do mathematics
- Humans think mathematically
- Mathematics is an extension of human logic which everyone has

Problems with mathematics means that:

- You missed a point somewhere and are now lost
- You never bother to revisit the section where you got lost

0.3 Learning skills

1. Quiet place(NO distractions) ☑
2. Proper desk ☑
3. Enough light ☑
4. Tidy environment ☑
5. Working mood ☑
6. Understanding ☑
7. Repetition ☑
8. Making links ☑

For tests/exams:
Predict what will appear
Think about how to answer the questions

Understanding
making your brain believe
that something is so important
that it accepts it as being true

☞ Use repetition to move information from short-term to long-term memory

Sleep enough

Stay fit and healthy

1 Using number systems

1.1 HCF — *Highest Common Factor*

Factors of 12: 1; 2; 3; $\boxed{4}$; 6; 12
Factors of 8: 1; 2; $\boxed{4}$; 8

➔ $HCF = 4$

1.2 LCM — *Lowest Common Multiple*

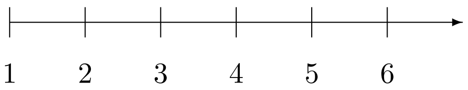
Multiples of 5: 5; 10; 15; $\boxed{20}$; 25; 30; 35; ...
Multiples of 4: 4; 8; 12; 16; $\boxed{20}$; 24; 28; ...

➔ $LCM = 20$

1.3 Examples of number systems

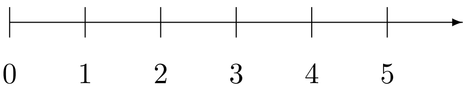
1.3.1 Natural Numbers

$\mathbb{N}$:



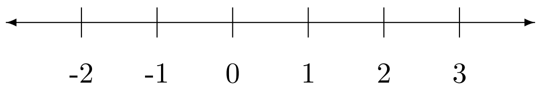
1.3.2 Whole Numbers

$\mathbb{N}_0$:



1.3.3 Integers

$\mathbb{Z}$:



1.4 Division by zero

Division by zero is undefined!!!!
Example: $10 \div 0 =$ *Undefined*

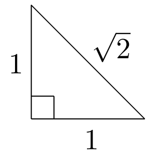
1.5 Rational numbers

A number is **rational** ($\mathbb{Q}$) if it

- $\boxed{\text{is in the form } \frac{a}{b}\text{, where } a \text{ and } b \text{ are integers and } b \neq 0.}$
- is a **terminating** decimal (*a decimal that stops after a number of decimal places* $\rightarrow \frac{47}{8} = 5.875$).
- is a **recurring** number (*Example:* $\frac{1}{3} = 0.33333\ldots = 0.\dot{3}$).

1.6 Irrational numbers ($\mathbb{Q}'$)

Examples: $\sqrt{30}$, π, $\sqrt{2}$
Cannot be written as a fraction of two integers.

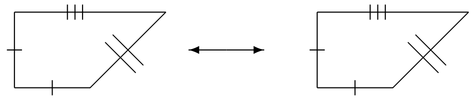


☺ Chapter done!!! ☺

2 Looking at polygons and polyhedra

2.1 Congruent polygons

Congruent polygons are identical in *size* and *shape*.



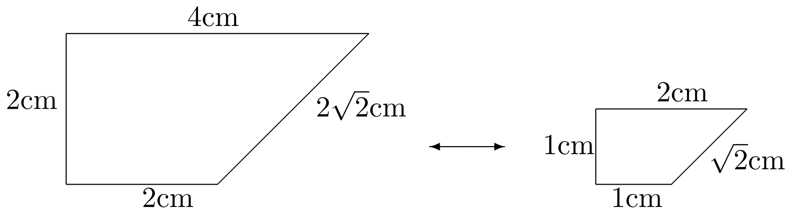
Two triangles are congruent if:

1. 3 sides of both triangles are equal (**SSS**)
2. 2 sides and included angle of both triangles are equal (**SAS**)
3. 2 angles and one corresponding side of both triangles are equal (**AAS**)
4. hypotenuse and one side of two right-angled triangles are equal (**HS**)

LEARN THIS!!!

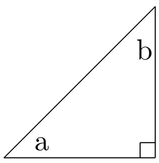
2.2 Similar polygons

Similar polygons contain the same *angles* and the length of their sides are *proportional* to one another.



2.3 Angles in a triangle

LEARN THIS!!! **The angles in any triangle add up to 180^0.** **LEARN THIS!!!**

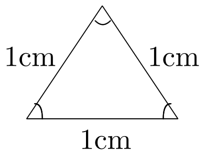


$a + b + 90^0 = 180^0$

2.4 Equilateral triangles

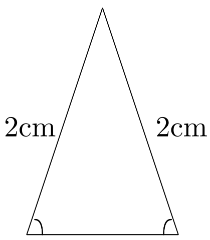
Triangles with all three angles equal (and all three sides equal) **are called *equilateral triangles*.**

All angles $= 60^0$



2.5 Isosceles triangles

Triangles with two angles equal (and hence the sides opposite those angles equal) **are called *isosceles triangles*.**



2.6 Polyhedra

V Vertices

F Faces

E Edges

LEARN THIS!!! ☞ $\boxed{\mathbf{V+F-E=2}}$ ☜ **LEARN THIS!!!**

2.6.1 Prisms

Prism A polyhedron in which two congruent, parallel polygons have their edges joined by parallelograms.

Right Prism has lateral faces which are rectangles.

2.6.2 Examples of shapes

1. Parallelogram
2. Rectangle
3. Kite
4. Isosceles trapezium
5. Rhombus
6. Square

☺ Chapter done!!! ☺

3 Measuring with accuracy △

3.1 Converting between Fahrenheit and Celsius

$\boxed{F = C \cdot \frac{9}{5} + 32}$ $\boxed{C = \frac{5}{9}(F - 32)}$

3.2 Area and volume

Area of **square** of side **l** cm	l^2 cm^2
Area of **rectangle** of **x** cm by **y** cm	xy cm^2
Area of **circle** of radius **r** cm	πr^2 cm^2
Area of **triangle** with base **b** cm and height **h** cm	$\frac{1}{2}bh$ cm^2
Area of **parallelogram** with base **b** cm and height **h** cm	bh cm^2
Volume of **cube** with side **l** cm	l^3 cm^3
Volume of **rectangular solid** of **x** cm by **y** cm by **z** cm	xyz cm^3

☜ **LEARN THIS!!!**

3.2.1 Volume calculations

Basic Rule:

Volume of a **cylinder**	area of circular base × height
Volume of a **right prism**	area of polygonal base × height

☜ **LEARN THIS!!!**

☺ Chapter done!!! ☺

4 Exponents

4.1 Definitions

$$\left(\frac{1}{3}\right)^3 = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}$$

$$\left(\frac{1}{3}\right)^{-1} = 3$$

$\left(\frac{1}{2}\right)^3$ is called a **Power**. ☞	$()^3$	is the **Index/Exponent**
	$\frac{1}{2}$	is the **Base**

4.2 Power summary

LEARN THIS!!! ☞

$$a^m \times a^n = a^{m+n}$$
$$(a^m)^n = a^{mn}$$
$$a^m \div a^n = a^{m-n}$$
$$\frac{1}{a^m} = a^{-m}$$
$$a^0 = 1$$

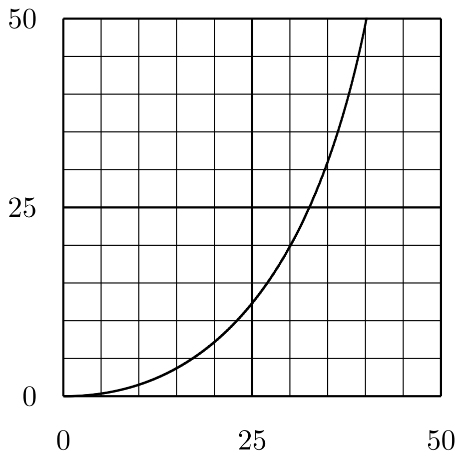
☜ **LEARN THIS!!!**

4.3 Scientific power

0.1	=	$\frac{1}{10}$	=	10^{-1}	=	one-tenth
0.01	=	$\left(\frac{1}{10}\right)^2$	=	10^{-2}	=	one-hundredth
0.001	=	$\left(\frac{1}{10}\right)^3$	=	10^{-3}	=	one-thousandth
0.0001	=	$\left(\frac{1}{10}\right)^4$	=	10^{-4}	=	one-ten thousandth
0.00001	=	$\left(\frac{1}{10}\right)^5$	=	10^{-5}	=	one-hundred thousandth
0.000001	=	$\left(\frac{1}{10}\right)^6$	=	10^{-6}	=	one-millionth

4.4 Scientific prefixes

Power	**Prefix**
10^{-12}	pico
10^{-9}	nano
10^{-6}	micro
10^{-3}	milli
10^{-2}	*centi*
10^{-1}	*deci*
10^{1}	*deca*
10^{2}	*hecto*
10^{3}	kilo
10^{6}	mega
10^{9}	giga
10^{12}	tera



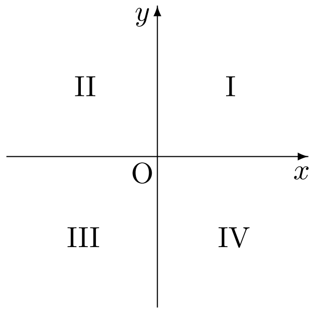
4.5 Surds

$$\sqrt[3]{1000} = (1000)^{\frac{1}{3}} \quad \rightarrow \quad \sqrt[a]{b} = (b)^{\frac{1}{a}}$$

$\sqrt{\ }$ and $\sqrt[3]{\ }$ are called **surd** signs

☺ Chapter done!!! ☺

5 Graphs



People use graphs to:

- show real-life examples (stockexchange)
- to communicate mathematical ideas
- to solve problems

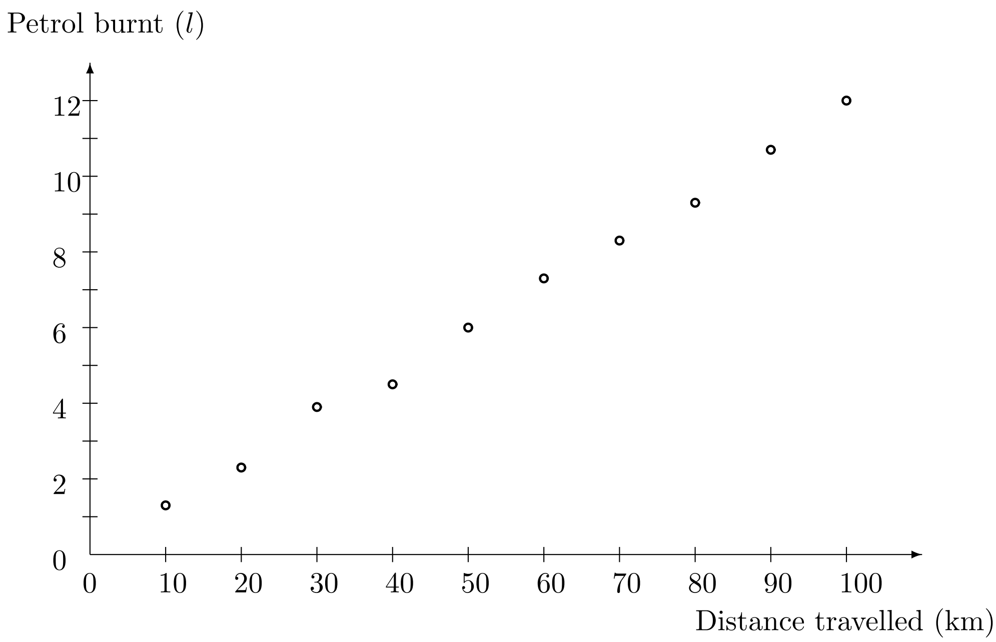
5.1 Cartesian plane

Important things to remember:

Coordinate axes	The number lines used
Origin	The point (0;0) where the axes cross
x-axis	Usually the horizontal axis
y-axis	Usually the vertical axis
Quadrants	Four quarters of the Cartesian plane divided up by the axes
x-coordinate	Usually the first number in an ordered pair (**abscissa**)
y-coordinate	Usually the second number in an ordered pair (**ordinate**)
Coordinates of a point	The ordered pair consisting of an x-coordinate and a y-coordinate

5.2 Plotting point graphs — Petrol consumption of Mr Scriba's Polo Classic

x	y
10	1.3
20	2.3
30	3.9
40	4.5
50	6.0
60	7.3
70	8.3
80	9.3
90	10.7
100	12.0



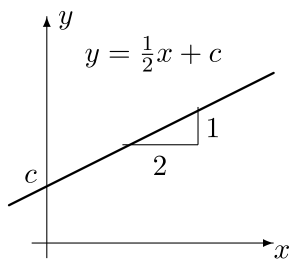
5.2.1 Gradient

LEARN THIS! ☞ $\boxed{\text{Gradient} = \frac{\mathbf{\Delta y}}{\mathbf{\Delta x}} = \frac{\mathbf{y_b - y_a}}{\mathbf{x_b - x_a}}}$ $\left(= \frac{12.0l - 1.3l}{100\text{km} - 10\text{km}} = \frac{10.7l}{90\text{km}} = 0.119l/\text{km} = 11.9l/100\text{km} \right)$

5.2.2 Straight line graphs

m	=	gradient
c	=	y-intercept

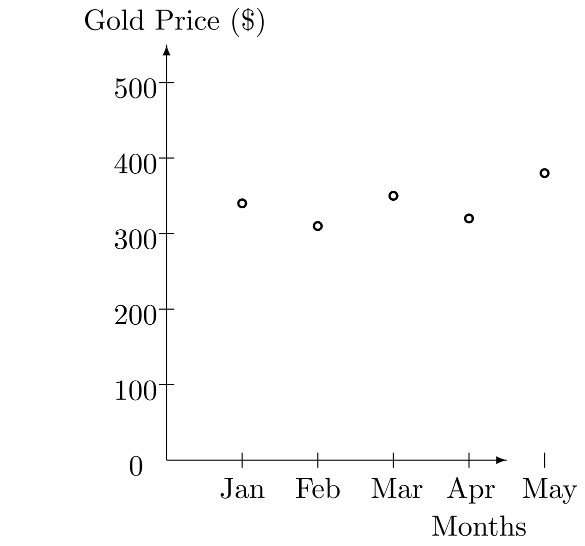
$$\mathbf{y = mx + c}$$

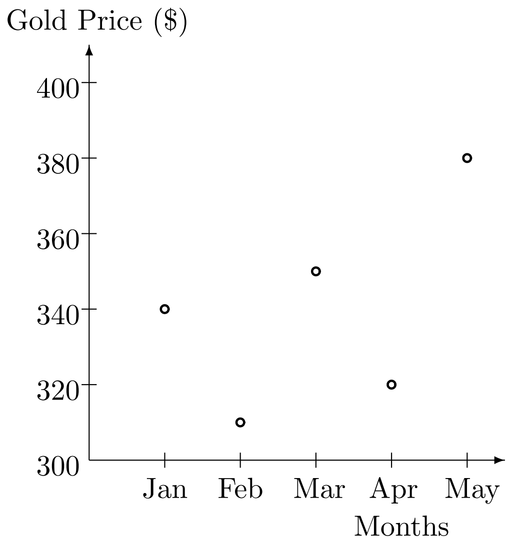


☺ Chapter done!!! ☺

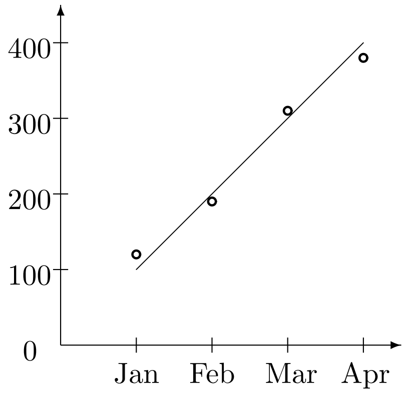
6 Statistics

6.1 Scale

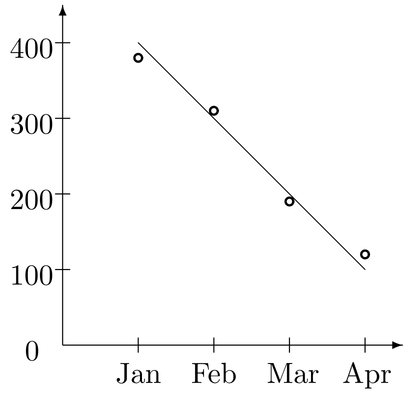




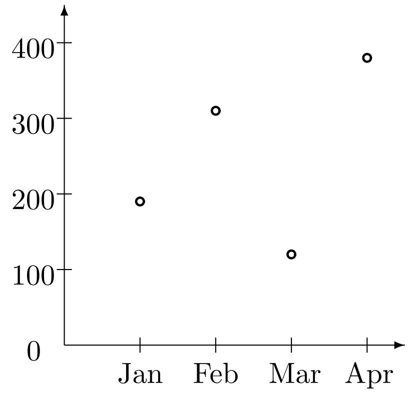
Positive correlation:



Negative correlation:



No correlation:



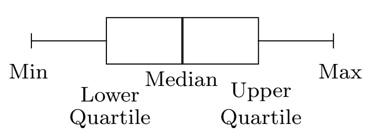
6.2 Statistical definitions

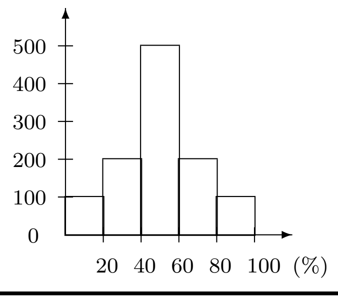
Mode	=	the value that appears most often in a set of data
Mean ($\overline{x}$)	=	$\left(\frac{\text{sum of all values}}{\text{number of values}}\right)$
Lower quartile	=	the value a quarter of the way through the ranked data
Median	=	the value halfway through a set of ranked data
Upper quartile	=	the value three quarters of the way through the ranked data
Counting data	=	data obtained by counting how many of a kind there are
Measurement data	=	data obtained by measuring with some instrument

☜ **LEARN THIS!!!**

6.3 Statistical plots

1. Box and whisker plot
2. Histogram
3. Stem-and-leaf plot





	1	6
7	2	8
11	3	2577
97661	4	11334568
988772	5	1234444578
88766321	6	335567
982	7	6
20	8	4
	9	2

Marks %

☺ Chapter done!!! ☺

7 Manipulating algebra

7.1 Algebraic definitions

Term	=	constants(numbers) and variables either **multiplied** or **divided** by each other
Like Terms	=	terms in the same power
Polynomial	=	a number of terms added or subtracted (the powers must be natural numbers)

$\sqrt{4x} = (4x)^{\frac{1}{2}}$ and $\frac{1}{x} = x^{-1}$
are not polynomials
➔ their powers are not natural numbers

- Like terms may be added or subtracted
- Unlike terms may not be added nor subtracted

$$2x^3 + 3x^{\boxed{2}} + 2x^{\boxed{2}} + x = 2x^3 + 5x^2 + x$$

- Terms are usually arranged in ascending or descending form

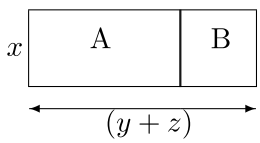
The **degree** of a polynomial is the highest exponent of the variable.

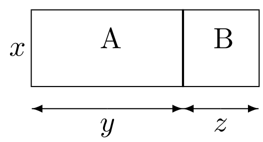
Monomial	=	has only one term	$4x^3$
Binomial	=	has two terms	$3x^2 + 5$
Trinomial	=	has three terms	$4x^2 - 2x + 1$

Equivalent expressions — Expressions that are equal for all values of their variables
Example: $(x+2)^2 \equiv x^2 + 4x + 4$

7.2 Distributive Law

$$x(y+z) = xy + xz$$





area of rectangles A and B = area of rectangle A + area of rectangle B

$$(a+b)(c+d) = ac + ad + bc + bd$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

7.3 Difference of two squares

LEARN THIS!!! $A^2 - B^2 = (A+B)(A-B)$ **LEARN THIS!!!**

7.4 Factorization method — How to factorize $x^2 + bx + c$

☟ **LEARN THIS!!!** ☟

- The coefficient of x^2 term must be 1 for the method to work
- If the sign of *term 3* is **"+"** — Look for two numbers for which
  1. the product is equal to *term 3*
  2. the sum is equal to *term 2*
  - The signs of the two factors will both be equal to the sign of *term 2*
- If the sign of *term 3* is **"-"** — Look for two numbers for which
  1. the product is equal to *term 3*
  2. the difference is equal to *term 2*
  - The signs of two factors must be different, with the sign of the larger number equal to the sign of *term 2*

Example 1:	$x^2 - 8x + 12$	➔ Numbers = 6,2	➔ $x^2 - 8x + 12 = (x-6)(x-2)$
Example 2:	$x^2 - x - 12$	➔ Numbers = 4,3	➔ $x^2 - x - 12 = (x-4)(x+3)$

Check your work by multiplying out the answer

From example 2: $(x-4)(x+3) = x^2 + 3x - 4x - 12 = x^2 - x - 12$

7.5 General rules

$\frac{6}{15} = \frac{2}{5} \times \frac{3}{3} = \frac{2}{5}$

$\frac{5\times 2}{5\times 3} = \frac{2}{3}$ ☜ **BUT** ☞ $\frac{5+2}{5+3} = \frac{7}{8} \neq \frac{2}{3}$

To divide by a fraction, invert and multiply ➔ $\frac{6}{8} \div \frac{3}{2} = \frac{6}{8} \times \frac{2}{3} = \frac{2}{4} = \frac{1}{2}$

$a \pm 0 = a$

$a \times 0 = 0$

$\frac{a}{0}$ or $a \div 0$ is meaningless ☞ **Division by 0 is undefined**

☜ **LEARN THIS!!!**

7.6 Adding and subtracting fractions

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}$$

Rule: To add or subtract two fractions, the denominators must be of the same order

LEARN THIS!!! ☞

$$\frac{a}{b} + \frac{c}{d} = \left(\frac{a}{b} \times \frac{d}{d}\right) + \left(\frac{c}{d} \times \frac{b}{b}\right) = \frac{ad + cb}{bd}$$

☜ **LEARN THIS!!!**

Example — Rewrite the following as a single fraction:

$$\frac{2}{3x^2} + \frac{1}{2xy} - \frac{5}{6y}$$

☞ Lowest Common Denominator (LCD) $= 6x^2y$

$$\frac{2}{3x^2} + \frac{1}{2xy} - \frac{5}{6y} = \left(\frac{2}{3x^2} \times \frac{2y}{2y}\right) + \left(\frac{1}{2xy} \times \frac{3x}{3x}\right) - \left(\frac{5}{6y} \times \frac{x^2}{x^2}\right) = \frac{4y + 3x - 5x^2}{6x^2y}$$

☺ Chapter done!!! ☺

8 Graphing straight lines

8.0.1 The essence of straight line graphs (know this in your sleep):

!!SLEEP MATERIAL!! ☞ $\mathbf{y = mx + c}$ ☜ **!!SLEEP MATERIAL!!**

!!SLEEP MATERIAL!! ☞

m = gradient
c = y-intercept

☜ **!!SLEEP MATERIAL!!**

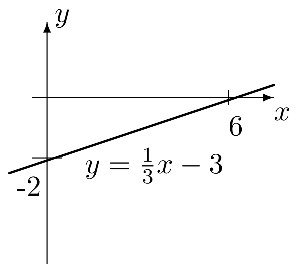
8.1 Plotting techniques

8.1.1 x-y intercept method

- Find y-intercept by setting $x = 0$
- Find x-intercept by setting $y = 0$

☜ **LEARN THIS!!!**

- Plot the two points and draw a straight line through them



Example: $y = \frac{1}{3}x - 2$

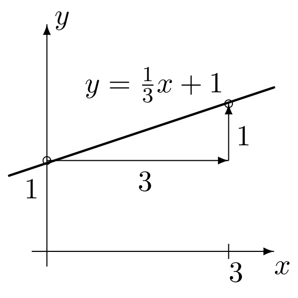
Set x = 0: $y = \frac{1}{3}0 - 2 = -2$
Set y = 0: $y = \frac{1}{3}x - 2 = 0$ ➔ $\frac{1}{3}x = 2$ ➔ $x = 2 \cdot 3 = 6$

8.1.2 Gradient method

- Find y-intercept by setting $x = 0$
- Plot y-intercept

☜ **LEARN THIS!!!**

- Gradient $m = \frac{\Delta y}{\Delta x}$ ☞ travel Δx units to the right and Δy units up.
- Plot second point and connect points with a straight line.



Example: $y = \frac{1}{3}x + 1$

Set x = 0: $y = \frac{1}{3}0 + 1 = 1$
Gradient $\mathbf{m = \frac{\Delta y}{\Delta x} = \frac{1}{3}}$ ➔ $\Delta x = 3$ and $\Delta y = 1$

8.1.3 Substitution plot

- Choose ten values of x along the x-axis

☜ **LEARN THIS!!!**

- Substitute each value into the equation and calculate the appropriate y-coordinate
- Plot the 10 points and connect them with a straight line

☺ Chapter done!!! ☺

9 Solving equations and inequalities

9.1 Solving basic equations

9.1.1 The problem:

Find the value of x so that $\mathbf{3x+2}$ and $\mathbf{x+6}$ are equal.

Solution 1: Make a table and start substituting values for x until $3x+2 = x+6$.

x	$3x+2$	$x+6$
1	5	7
2	$\boxed{8}$	$\boxed{8}$
3	11	9
4	14	10

Problem: Too slow!!!

Solution 2: Set the two expressions equal to one another and solve for x:

$$\begin{aligned} 3x+2 &= x+6 && \textit{(Subtract } x \textit{ and 2 from both sides)} \\ 3x-x &= 6-2 && \textit{(Add together the } x \textit{ and the constants)} \\ 2x &= 4 && \textit{(Multiply both sides by } \tfrac{1}{2}\textit{)} \\ x &= 2 \end{aligned}$$

Note: Move all terms containing x to the one side of the equation and all constants to the other.

!Always! ☞ Check your solution by substituting back into the original expressions. **They should be equal.** ☜ **!Always!**

9.1.2 A problem with fractions:

Solve $\mathbf{\frac{3x}{2} + \frac{2}{3} = \frac{1}{2} + x + \frac{7}{6}}$ for x.

Solution — multiply through by the *Lowest Common Denominator* (LCD) and solve as before:

$$\begin{aligned} \frac{3x}{2}+\frac{2}{3} &= \frac{1}{2}+x+\frac{7}{6} && \textit{(Multiply both sides by LCD = 6)} \\ 6\cdot\left(\frac{3x}{2}+\frac{2}{3}\right) &= 6\cdot\left(\frac{1}{2}+x+\frac{7}{6}\right) && \textit{(Simplify each term)} \\ 9x+4 &= 3+6x+7 && \textit{(Separate } x \textit{ terms and constants)} \\ 9x-6x &= 3+7-4 && \textit{(Simplify)} \\ 3x &= 6 && \textit{(Multiply both sides by } \tfrac{1}{3}\textit{)} \\ x &= 2 \end{aligned}$$

!Always! ☞ Check your solution by substituting back into the original equation. **LHS should equal RHS.** ☜ **!Always!**

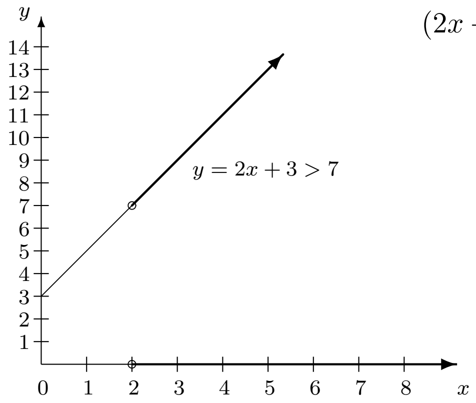
9.2 Inequalities

9.2.1 Problem:

Solve $\mathbf{2x} + \mathbf{3} > \mathbf{7}$ for x:

Solution:

$$\begin{aligned} 2x+3 &> 7 && \textit{(Subtract 3 from both sides)} \\ (2x+3)-3 &> (7)-3 && \textit{(Simplify)} \\ 2x &> 4 && \textit{(Multiply by } \tfrac{1}{2}\textit{)} \\ x &> 2 \end{aligned}$$



9.2.2 Inequality rules

1.	$+ve$ or $-ve$ number added or subtracted to both sides	no change
2.	multiplied or divided through by $+ve$ number	no change
3.	multiplied or divided through by $-ve$ number	**reverse sign!**

☜ **LEARN THIS!!!**

Examples:

1.	$x+2>3$	➔	$x>1$	*(Subtract 2 from both sides)*
2.	$2x>4$	➔	$x>2$	*(Divide both sides by 2)*
3.	$-4x>12$	➔	$x<-3$	*(Divide both sides by -4)*

9.3 Simultaneous equations

9.3.1 Problem:

Solve for x and y:

$$\left.\begin{aligned} 3(x-1) &= x+5 \\ 2x+5 &= y-x+2 \end{aligned}\right\}$$

9.3.2 Rule:

1.	Solve for either x or y in the first equation
2.	Substitute result into second equation

☜ **Remember this!**

Solution:

$$\begin{aligned} 3(x-1) &= x+5 && \ldots① \\ 2x+5 &= y-x+2 && \ldots② \\ \text{From ①:} \quad 3x-3 &= x+5 \\ \therefore\ 2x &= 8 \\ \therefore\ x &= 4 \\ \text{Substitute into ②:} \quad 2(4)+5 &= y-4+2 \\ \therefore\ 13 &= y-2 \\ \therefore\ y &= 15 \end{aligned}$$

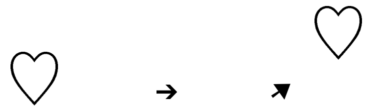
☺ Chapter done!!! ☺

10 Transforming space

10.1 Possible transformations

The following are possible:

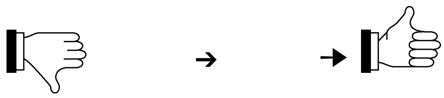
1. **Translation:**



2. **Reflection of one shape:**

3. **Rotation through a given angle:**

4. **Glide reflection:**



10.2 Translation

- If two shapes differ by a translation, they are congruent.
- In a translation, the line segments joining the corresponding points are parallel to one another, and they are all the same length.
- In a translation all points move.

If two patterns differ by a translation and $(x; y)$ is the original point, the position of its image will be at $(x + a; y + b)$. (a and b can be positive or negative)

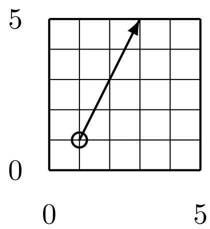
Column vectors:

A column vector contains two numbers.

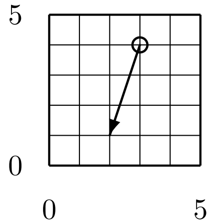
- The top number tells us how much sideways movement there is
- The bottom number tells us how much up or down movement there is

For example:

$\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ represents a move 2 units to the right
represents a move 4 units up



$\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ represents a move 1 unit to the left
represents a move 3 units down



10.3 Reflection

The line about which a shape is folded to give a reflection is called a **line of reflection** or an **axis of symmetry**.

10.4 Rotation

- Turning motions are called **rotations**.
- A rotation can be either clockwise ↻ or anticlockwise ↺.
- In a rotation there is always one point that doesn't change position is called the **centre of rotation**.
- If two figures differ by a rotation, then corresponding points are the same distance from the centre of rotation.
- The **centre of rotation** is the only point that does not move during a rotation and is therefore also called the **invariant point**.

☺ Chapter done!!! ☺

11 Working with ratio, rate and proportion

11.1 Ratio

The Rule: When dealing with ratios, we must compare quantities that have the same units.

!LEARN THIS!

— When **increasing** an amount in a given ratio,
your answer should be **greater than** the amount you started with.
—When **decreasing** an amount in a given ratio,
your answer should be **less than** the amount you started with.

Example 1: Increase 15 in the ratio 5 : 6:

$$15 \div \frac{5}{6} = 15 \times \frac{6}{5} = 3 \times 6 = 18$$

Example 2: Decrease 24 in the ratio 3 : 2:

$$24 \div \frac{3}{2} = 24 \times \frac{2}{3} = 8 \times 2 = 16$$

11.2 Rate

The Rule: When we compare quantities with different units, we talk of rates.

Example: To get to university I have to walk 2km, which takes me $\frac{1}{2}$ an hour. At what **rate** do I walk?

Solution:

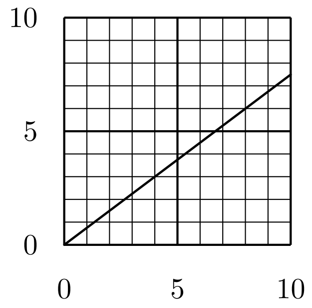
$$\text{Rate} = \frac{\text{Distance}}{\text{Time}} = \frac{2\text{km}}{1/2\text{hr}} = 4\text{km/hr}$$

11.3 Proportion

If numbers increase/decrease in **direct proportion**
they have a **constant quotient**.

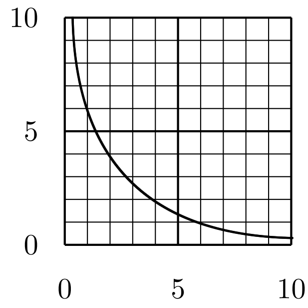
If numbers increase/decrease in **inverse proportion**
they have a **constant product**.

Examples that do not contain a constant quotient nor product are **not in proportion**.



A graph of quantities
that are in proportion
is a straight-line graph
and passes through the origin.

$y = mx + 0$

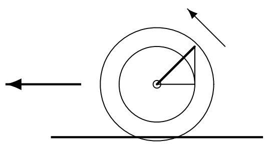


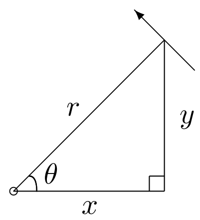
A graph of quantities
that are inversely proportional is a hyperbola.
An increase on one axis
causes a decrease on the other axis.

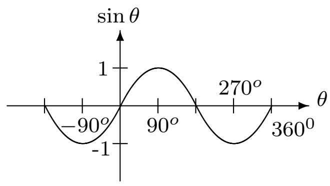
$y = \frac{k}{x}$

☺ Chapter done!!! ☺

12 Applying trigonometry







!!SLEEP MATERIAL!! ☞

$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$

$\tan\theta = \frac{y}{x}$

☜ **!!SLEEP MATERIAL!!**

☺ Chapter done!!! ☺

13 Mathematisning finance ($ THE MONEY $)

13.1 Exchange rates

- The exchange rate indicates the strength or weakness of the rand compared to the currency of another country
- Exchange rates fluctuate on a daily basis and are reported on radio stations, TV, the Internet, and in newspapers and magazines.
- The exchange rate has a direct impact on the cost of goods that are imported into the country.

13.2 Simple interest

Interest is paid only on the initial capital invested over the total period of time

Simple interest is often used in hire purchase agreements (buyer pays goods off over a certain period).

!LEARN THIS! ☞ **Total amount of simple interest** $= i \times P \times n.$ ☜ **!LEARN THIS!**

!LEARN THIS! ☞ **Total amount to pay:** $\mathbf{A = P + (i \times P \times n)}$ ☜ **!LEARN THIS!**

- A = final amount to pay at the end of the investment or loan period.
- P = initial amount originally invested or borrowed
- i = interest rate per annum (year) $= \frac{r}{100}$ — Example: $i = 0.12 \rightarrow r = 12\%$ ☜ **WATCH OUT!**
- n = number of years

13.3 Compound interest

At the end of each period, the interest is added to the capital
and interest is paid on the initial capital
AND the interest accumulated at the end of each period

LEARN THIS!!! ☞ **Total amount to pay:** $\mathbf{A = P(1 + i)^n}$ ☜ **LEARN THIS!!!**

- A = final amount to pay at the end of the investment or loan period.
- P = initial amount originally invested or borrowed
- i = interest rate per annum (year) $= \frac{r}{100}$ ☜ **WATCH OUT!**
- n = number of years

13.4 Inflation rates

- Inflation is compounded annually and calculated on the previous year's prices. The compound interest formula can be used to calculate the prices of goods over a period of time using an average rate of inflation.

13.5 Depreciation rates

Depreciation is the loss of assets, machinery or equipment through age or use.

There are two ways of calculating depreciation:

Straight-line depreciation	=	the depreciation rate is calculated as a percentage of the **original value** of the asset each year.
Reducing balance depreciation	=	the depreciation rate is calculated as a percentage of the **reduced value** of the asset each year.

- Straight-line depreciation will result in the equipment having no value after a period of time
- Depreciation on a reducing balance will ensure that equipment always has some value at the end of a certain period

13.5.1 Depreciation on a reducing balance:

LEARN THIS!!! ☞ $\mathbf{A = P(1 - i)^n}$ ☜ **LEARN THIS!!!**

- A = future value of equipment
- P = present value of equipment
- i = annual rate of depreciation
- n = number of years

13.6 Compound interest over other periods of time

Interest is often compounded quarterly, monthly, or even daily, instead of annually. The equation stays the same:

$\mathbf{A = P(1 + i)^n}$ ☜ **Note: '+' instead of '-'!**

But i and n change:

— i	=	interest rate per compounding period
— n	=	number of compounding periods

☜ **REALLY IMPORTANT!**

Example 1: Interest rate of 12% is compounded quarterly.

- $i = \frac{0.12}{4} = 0.03$
- $n = \text{years} \times 4$

Example 2: Interest rate of 12% is compounded daily.

- $i = \frac{0.12}{365} = 0.00328$
- $n = \text{years} \times 365$

☺ Chapter done!!! ☺

14 Counting the chances

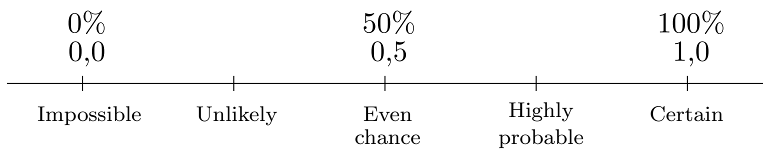
Statistics started because of gambling

14.1 Statistical definitions

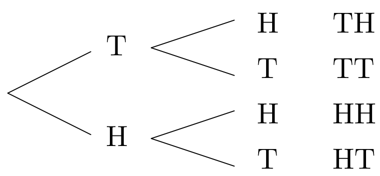
Trial	=	an action, such as spinning a coin
Outcome	=	a result of a trial (getting a 3 when rolling a dice)
Event	=	a clearly defined set of outcomes (getting an even number when rolling a dice)
Experimental probability	=	$\frac{\text{Number of times the event occured}}{\text{Total number of trials}}$
Theoretical probability	=	$\frac{\text{Number of ways the event can occur}}{\text{Total number of possible outcomes}}$

LEARN!

- The theoretical probability can only be calculated without doing an experiment in cases where all the possible outcomes of a trial are equally likely.
- The experimental probability gets closer to the theoretical probability the greater the number of trials.
- Probability values range between 0 and 1. These values can be arranged on a probability scale:



- A tree diagram can assist in working out the possible outcomes in the case of combined events.



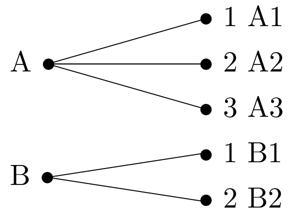
- In cases where equally likely events involve areas, the probability is calculated as

$$\text{Probability} = \frac{\text{Success area}}{\text{Total area}}$$

14.2 Counting principles

14.2.1 The addition principle

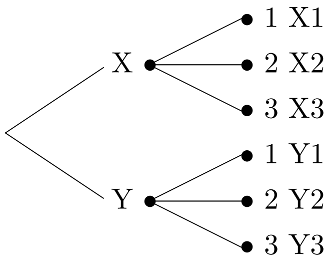
If the number of choices exclude each other we add them up.



➔ Total number of possibilities $= 3 + 2 = 5$

14.2.2 The multiplication principle

If a number of choices can be made and then for each of the first choices a second lot of choices can be made, then the total number of choices that are possible is obtained by multiplication.



➔ Total number of possibilities $= 2 \times 3 = 6$

14.3 Probability trees

14.3.1 Independent events

- Two events are independent if the occurrence of one of the events (P) **does not affect the probability** that the other (e.g. Q)will occur.
- For independent events P and Q, the probability of getting P and Q is the product of the probability of getting P with the probability of getting Q.

For independent events P and Q:

Probability of P **AND** Q = (Probability of P) × (Probability of Q) ☜ **LEARN THIS!!!**

14.3.2 Mutually exclusive events

- Two events are mutually exclusive if there are **no outcomes in common** to the two.
- For mutually exclusive events P and Q, the probability of getting P and Q is the sum of the probability of getting P and the probability of getting Q.

For mutually exclusive events P and Q:

Probability of P **OR** Q = (Probability of P) + (Probability of Q) ☜ **LEARN THIS!!!**

14.3.3 Complementary events

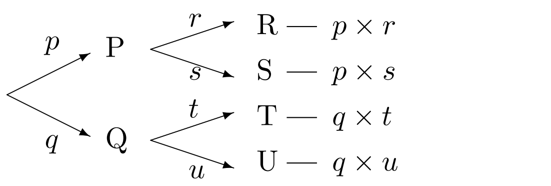
- Event P is complementary to event Q if they are **mutually exclusive** and if they together provide **all possible outcomes**.

For complementary events P and Q:

Probability of $P = 1-$ (Probability of Q) ☜ **LEARN THIS!!!**

14.3.4 Probability trees

- We multiply the probabilities when considering events going along a path.
- We add the probabilities when dealing with the probability of going along several paths.



Total: $pr + ps + qt + qu$

☺ Chapter done!!! ☺

15 Solving problems

15.1 Problem solving steps

1.	Understand the problem
2.	Explore the problem
3.	Formulate a conjecture
4.	Extend the problem

☜ **IMPORTANT!!!**

15.1.1 Understand the problem

Explain the problem to other people. You only understand the problem if you can explain it to someone else.

15.1.2 Explore the problem

Create a picture or draw a diagram. A picture helps you to visualise the problem. "A picture is worth a thousand words".

15.1.3 Formulate a conjecture

Keep the problem as simple as possible and use the previous two steps to find a solution to the problem.

15.1.4 Extend the problem

Use the solution to solve more complicated problems. Be sure to understand such complex problems first, by applying steps 1 and 2 again.

15.2 Look for a pattern

- Looking for patterns often enables us to formulate a conjecture. Once a pattern has been established, it is possible to generalise.
- A pattern indicates **order** and enables us to predict an **outcome**.

☺ Chapter done!!! ☺

THE END

By covering the contents of this document,
you have successfully completed the Grade 9 syllabus!

Well done!

☞ !!Now practice as many examples in your textbook as possible!! ☜

$\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$

$\div$

$\times$

Grade 9 Mathematics Made Fun[1]

with Stefan Scriba

$\text{Area} = \pi r^2$

$y = ax^2 + bx + c$

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[1] This tutorial is based on the book *Outcomes-based Classroom Mathematics — Grade 9*, Laridon *et al.*, Heinemann Publishers (Pty) Ltd, 2001, ISBN 1 86853 990 3 and all credit is given to the authors of the book. This document is not intended to replace the book, but should be used in conjunction with the book.