Calculus Summary

Stefan Scriba

This document was produced in LaTeX— try and beat this, MSWord!

February 19, 2003

1 The nightmare: Trigonometric Functions

Hint: *Learn the following by heart!* :)

1.1 Identities

$$\begin{aligned}
\sin^2 x + \cos^2 x &= 1 \\
\sec^2 x &= 1 + \tan^2 x \\
\csc^2 x &= 1 + \cot^2 x
\end{aligned}$$

$$\begin{aligned}
\sin(x+y) &= \sin x \cdot \cos y + \cos x \cdot \sin y \\
\cos(x+y) &= \cos x \cdot \cos y - \sin x \cdot \sin y
\end{aligned}$$

$$\begin{aligned}
\sin(-x) &= -\sin x \\
\cos(-x) &= \cos x \\
\sin\left(\frac{\pi}{2} - x\right) &= \cos x \\
\cos\left(\frac{\pi}{2} - x\right) &= \sin x
\end{aligned}$$

$$\begin{aligned}
\sin 2x &= 2\sin x \cdot \cos x \\
\cos 2x &= \cos^2 x - \sin^2 x \\
&= 2\cos^2 x - 1 \\
&= 1 - 2\sin^2 x
\end{aligned}$$

$$\begin{aligned}
\sin^2 x &= \frac{1 - \cos 2x}{2} \\
\cos^2 x &= \frac{1 + \cos 2x}{2}
\end{aligned}$$

$$\begin{aligned}
\sin x \cdot \cos y &= \frac{1}{2}\left[\sin\left(x+y\right)+\sin\left(x-y\right)\right] \\
\sin x \cdot \sin y &= \frac{1}{2}\left[\cos\left(x-y\right)-\cos\left(x+y\right)\right] \\
\cos x \cdot \cos y &= \frac{1}{2}\left[\cos\left(x+y\right)+\cos\left(x-y\right)\right]
\end{aligned}$$

1.2 Derivation

$$\begin{aligned}
\frac{d}{dx}\sin x &= \cos x \\
\frac{d}{dx}\cos x &= -\sin x \\
\frac{d}{dx}\tan x &= \sec^2 x \\
\frac{d}{dx}\cot x &= -\csc^2 x \\
\frac{d}{dx}\sec x &= \sec x \cdot \tan x \\
\frac{d}{dx}\csc x &= -\csc x \cdot \cot x
\end{aligned}$$

1.3 Derivatives of Inverses

$$\begin{array}{rclcrcl}
\frac{d}{dx}\arctan x & = & \frac{1}{1+x^2} & \Rightarrow & \int \frac{1}{1+x^2}\,dx & = & \arctan x \\
\frac{d}{dx}\arcsin x & = & \frac{1}{\sqrt{1-x^2}} & \Rightarrow & \int \frac{1}{\sqrt{1-x^2}}\,dx & = & \arcsin x
\end{array}$$

1.4 Integration

$$\begin{aligned}
\int \tan x \; dx &= \int \frac{\sin x}{\cos x}\; dx = -\log\left|\cos x\right| \\
\int \cot x \; dx &= \int \frac{\cos x}{\sin x}\; dx = \log\left|\sin x\right| \\
\int \sec x \; dx &= \log\left|\sec x+\tan x\right| \\
\int \csc x \; dx &= -\log\left|\csc x+\cot x\right|
\end{aligned}$$

This document will be further extended in due time—this should be helpful already!